Trigonometry Training Wheels

Understanding by Design

University of Mississippi School of Education

Tatum Hancock
# Unit Cover Page

**Unit Title:** Trigonometry Training Wheels  
**Grade Level:** 9th-12th Trigonometry

**Subject/Topic Areas:** Trigonometry with English Language Arts Integrated

**Key Words:** Sine, Cosine, Tangent, Unit Circle, Special Right Triangles Periodic, Radian Measure, Arc Length, Angle Measure, Radians, Degrees, Radius, Terminal Side, Theta, Proportion, Ratio, Oral Presentation

**Designed By:** Tatum Hancock  
**Length of Unit:** 8 days

**School District:** Oxford School District  
**School:** Oxford High School

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**Brief Summary of Unit:**
The purpose of this unit is to have students develop discovery learning skills while building fundamental trigonometric knowledge. The unit will cover the unit circle and students will be taught how to calculate \( \sin \theta \), \( \cos \theta \), and \( \tan \theta \) for given points on the unit circle. Students will be taught how to extend this concept to other right triangles with measurements of real numbers. The unit covers the definition of a radian to and takes to a level of deeper understanding and application. The unit also covers the basic idea of periodic functions, mostly focusing on the graph of a periodic function as opposed to the actual function itself. From there, the unit covers converting radians to degrees, and vice versa. The unit ends going in the direction of trigonometric applications for word problems. This unit is an introduction to trigonometry.

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**List and attach Print Materials/Resources**

**List and attach Internet Resources/Links**

- “Unit Circle Opener” Google Poll - [http://goo.gl/forms/H1R5gcykHy](http://goo.gl/forms/H1R5gcykHy)
- Handout – “Interactive Unit Circle”
- TI Inspire software (installed before the class, preferably at beginning of school year)
- TI Inspire Radian Measure Worksheet – teacher version
- TI Inspire Radian Measure Worksheet – student version
- Radian Measure.tns file
- “A Slice of Pi” Google Poll Homework - [http://goo.gl/forms/jO6ZxJ7vFO](http://goo.gl/forms/jO6ZxJ7vFO)
- “Periodic Graphs” Google Poll - [http://goo.gl/forms/1IQZeySMNt](http://goo.gl/forms/1IQZeySMNt)
- Sine and Cosine Graph Tables handouts
• “Trig Functions and Other Right Triangles” Google Poll Opener - http://goo.gl/forms/qunp2nwnUh
• Triangle cut-outs
• Performance Task Rubric
• Test study guide
• Performance Task materials (brought by the students)
• Ladder word problem
• “Unit Circle, Trigonometric Functions, and Periodicity” Part 1
• “Unit Circle, Trigonometric Functions, and Periodicity” Part 2
• Test answer key

http://bie.org/object/document/9_12_presentation_rubric_ccss_aligned#
http://www.politico.com/2012-election/results/president/mississippi/
http://apps.washingtonpost.com/local/highschoolchallenge/schools/2014/oxford-oxford-ms/
http://www.corestandards.org/Math/
Contextual Information

Oxford High School

Tatum Hancock

1. Knowledge of characteristics of students

Use the spaces provided below to address indicated characteristics of your students.

- Age-Range, Gender, Total number of students
  - Mrs. Wise’s 4th period Pre-Calculus class at Oxford High School is made up of three 12th grade students, thirteen 11th grade students, eight 10th grade students, and one 9th grade student. The students range in age from 15 to 18. Around 980 students attend Oxford High School with 54% white, 38% African American, and 8% other including Hispanic/Latino and Asian/Pacific Islander. Throughout an average day, Mrs. Wise teaches 126 students, with two of her classes being ACT Prep classes. Her 4th period Pre-Calculus class has a total of 25 students. There are 18 boys and 7 girls.

- Achievement Levels (Remedial, Average, Advanced/ Accelerated, or specify range in percentiles or grade-equivalent)
  - After conducting a student interest survey, every single student stated that they plan to move on to college, and each student named at least one four year university they would prefer to attend. Mrs. Wise’s 4th period class is an advanced group of students. There are no students at risk of failing the class or their grade. The students in this class for the most part all receive A’s on their homework assignments. For tests and quizzes, Mrs. Wise’s students receive an average of a mid to high B. Mrs. Wise stated that she, as their teacher, would classify the majority of the class as advanced/accelerated.
  - Oxford High School in general has an average ACT score of 22.7 with a four-year graduation rate of 85.6%. Oxford High School was named one of America’s Most Challenging High Schools in 2014 by The Washington Post. 76% of students who attend Oxford High School end up attending a four-year college after graduation.

- Socio-Economic Description
  - In 2014, 52% of students at Oxford High School received free or reduced lunch. Just judging by Mrs. Wise’s 4th period, most students are able to wear brand names such as Patagonia, Polo Ralph Lauren, Nike, Underarmour and more. There do not seem to be any significant financial struggles, at least in 4th period.

- Typical Demeanor of Students
  - The students in Mrs. Wise’s 4th period class are respectful and calm in general, especially when necessary to classroom instruction. Mrs. Wise’s class is lead with
strong discipline and little room for discussion between classmates. There is an assigned seating chart that was implemented in all of Mrs. Wise’s pre-calculus classes at the beginning of the second semester due to behavior issues experienced in another period. Students do not have snacks or candy in the classroom as they are not allowed. Many students lean their heads on their hands and arms when taking notes, while others simply place their heads on their desks and close their eyes. Very few students speak up to answer questions, unless Mrs. Wise calls on them. The students as a whole do not seem to be intrigued by any of the content material in the average daily lesson. If anything, when a new subject is brought up in a lesson, the overall mood of the class becomes worried and stressed out.

- Typical Interest and Involvement of Students

![Oxford High School Sports Involvement Chart](image_url)
The students at Oxford High School are involved in numerous school and community clubs and organizations as shown above.

2. Knowledge of students’ varied approaches to learning (Include information from learning styles inventory)
   - From an observational standpoint, the majority of the class is neither solely visual nor solely auditory learners. Mrs. Wise’s main instruction technique is direct instruction where she stands at the front of the class with the Apple TV and uses screenshots of solutions to show them how to work a problem. This seems to lead most students to put their heads down on their desks and/or have many unanswered questions about the material. The students do not have many opportunities to critically think about the lesson before being directly told what to do and how to do it. After asking the students about their learning style preferences, around half of the class said they are kinesthetic learners which would correspond to the number of students involved in sports activities. About half of the class said they are a combination of more than one, the most being kinesthetic and visual. The minority learning style was a combination of visual and auditory, as modeled by Mrs. Wise’s direct instruction technique.

3. Knowledge of students’ skills and prior learning
   - Oxford High School’s course guidebook states that students are required to have a strong background in Algebra 2 and Geometry. However, there is no entrance test they have to pass to enroll. Since it is a Pre-AP course, parents must sign their child’s schedule acknowledging they understand the difficulty of the class.

4. Knowledge of community and school district (Include a description of the community and school district)
<table>
<thead>
<tr>
<th></th>
<th>Oxford</th>
<th>Mississippi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Household Size (people)</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Median Resident Age (years)</td>
<td>27</td>
<td>36</td>
</tr>
<tr>
<td>Est. Median Household Income (dollars)</td>
<td>$32,500</td>
<td>$37,000</td>
</tr>
<tr>
<td>Est. Median House/Condo Value (dollars)</td>
<td>$221,114</td>
<td>$99,800</td>
</tr>
<tr>
<td>2012 Presidential Election Republican Votes</td>
<td>57%</td>
<td>55%</td>
</tr>
<tr>
<td>2012 Presidential Election Democrat Votes</td>
<td>41%</td>
<td>43%</td>
</tr>
</tbody>
</table>

As outlined in the previous table, the city of Oxford’s cost of living (in terms of house/condo value) is more than double that of the state of Mississippi as a whole. Political views accurately reflect the rest of the state’s political views with the majority of voters Republican.

Oxford School District is located in Lafayette County, Mississippi. Lafayette County was founded in 1836, only 12 years before The University of Mississippi was founded in Oxford, MS, of which 24,000+ of the residents/contributors of Oxford attend. Eight major highways run through the county, and there is one national protected area. An estimated population for 2013 was 51,318 people. The following chart outlines the racial makeup of Lafayette County.
Oxford has become a popular tourist attraction due to the varying atmospheres from fine dining and shopping experiences to the rowdy sporting events at the University of Mississippi. Many of the people living in Oxford are students of the University and contribute to much of the city’s revenue. About 980 of the 20,800 residents of Oxford are enrolled at Oxford High School.

Sources:
http://www.politico.com/2012-election/results/president/mississippi/
http://apps.washingtonpost.com/local/highschoolchallenge/schools/2014/oxford-oxford-ms/
Stage 1 – Identify Desired Results
(Stage 1 completed once for the unit)

Goal: Identify overall goal (s) of the unit based on the Mississippi Curriculum Frameworks or Common Core Standards.

Mathematics: Algebra

Mathematical Practices

5. Use appropriate tools strategically.
7. Look for and make use of structure.

Mathematics: Trigonometry

1. Extend the domain of trigonometric functions using the unit circle.
   a. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $x$, $\pi + x$, and $2\pi - x$ in terms of their values for $x$, where $x$ is any real number. CCSS.Math.Content.HSF.TF.A.3
   b. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. CCSS.Math.Content.HSF.TF.A.2
   c. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. CCSS.Math.Content.HSF.TF.A.1

2. Define trigonometric ratios and solve problems involving right triangles
   a. CCSS.Math.Content.HSG.SRT.C.6
      Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
   b. CCSS.Math.Content.HSG.SRT.C.7
      Explain and use the relationship between the sine and cosine of complementary angles.
   c. CCSS.Math.Content.HSG.SRT.C.8
      Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

ELA:

1. Presentation of Knowledge and Ideas:
   a. CCSS.ELA-Literacy.SL.9-10.4
      Present information, findings, and supporting evidence clearly, concisely, and
logically such that listeners can follow the line of reasoning and the organization, development, substance, and style are appropriate to purpose, audience, and task.

b. **CCSS.ELA-Literacy.SL.9-10.5**

Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.

2. **Comprehension and Collaboration:**
   a. **CCSS.ELA-Literacy.SL.11-12.2**

Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

b. **CCSS.ELA-Literacy.SL.11-12.3**

Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, assessing the stance, premises, links among ideas, word choice, points of emphasis, and tone used.

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**What understandings are desired?**

**The students will understand:**

- How the unit circle works and why each coordinate pair has its specific value.
- That radian measure of an angle is the length of the arc on the unit circle subtended by the angle.
- The trigonometric functions and their properties can be used in real situations involving real numbers.

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**Daily objectives: What key knowledge and skills will students acquire as a result of this unit? What should learners be able to do as a result of such knowledge? Include integrated content areas from the Mississippi Curriculum Frameworks. Label objectives with the DOK level of learning.**

**The students will:**

**Day 1:**

- Identify the four quadrants of the Cartesian plane. (DOK 1)
- Distinguish in which quadrants x-values are positive or negative and y-values are positive and negative. (DOK 2)
- Construct a unit circle using special right triangles specific to the unit circle. (DOK 2)
- Develop a logical argument explaining the coordinate points of the unit circle formed by the length of the legs of special right triangles. (DOK 3)
- Label the x and y coordinates for the major points on the unit circle. (DOK 1)
- Geometrically calculate the values for sine, cosine, and tangent for \(\pi/3\), \(\pi/4\) and \(\pi/6\) using special right triangles. (DOK 1)
- Draw the conclusion that each coordinate point on the unit circle contains the sine and cosine for each angle in radian measure. (DOK 3)

**Day 2:**

- Tell the relationship between a central angle, the radius, and the arc length of a circle. (DOK 1)
- Recognize that changing the radius of a circle does not affect the proportions in the circle.
(DOK 1)
• Apply concepts to convert an angle measure from radians to degrees and vice versa. (DOK 4)
• Create a proportion that can be used to convert the measure of an angle from degrees to radians and vice versa. (DOK 4)
• Use appropriate tools strategically (DOK 1, Algebra CCSS Mathematical Practice).
• Look for and make use of structure (DOK 1, Algebra CCSS Mathematical Practice).

Day 3:
• Distinguish between periodic graphs and non-periodic graphs. (DOK 2)
• Interpret and categorize graphs into categories of periodic or not periodic. (DOK 2)
• Calculate the decimal values for each radian measure on the unit circle. (DOK 1)
• Construct a graph of sine and cosine of each radian measure on the unit circle. (DOK 4)

Day 4:
• Calculate the values of sine, cosine, and tangent for \( x, \pi + x, \) and \( 2\pi - x \) in terms of their values for \( x \), where \( x \) is any real number. (DOK 1)
• Relate the ratios \( \sin \theta = \frac{y}{r}, \cos \theta = \frac{x}{r}, \) and \( \tan \theta = \frac{y}{x} \) to any other right triangle with real numbers. (DOK 3)
• Interpret the ratios \( \sin \theta = \frac{y}{r}, \cos \theta = \frac{x}{r}, \) and \( \tan \theta = \frac{y}{x} \) to mean \( \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \) and \( \tan \theta = \frac{\text{opposite}}{\text{adjacent}} \) for other right triangles not on the unit circle. (DOK 3)

Day 5:
• Recall all material learned so far that will be included on the test. (DOK 1)
• Organize thoughts for completion of upcoming project. (DOK 2)

Day 6:
• Recall definitions. (DOK 1)
• Use concepts from unit to answer test questions for “Unit Circle, Trigonometric Functions, and Periodicity” Test. (DOK 1)

Days 7 & 8:
• Create a word problem. (DOK 4)
• Apply concepts of trigonometric functions and right triangles to real life situations. (DOK 4)
• Show these ideas in as a presentation to the class. (DOK 2)
**Stage 2 – Planning Assessment**
(Stage 2 completed once for the unit)

**Performance Task(s):** List the names of each performance task here and attach a copy of the entire assignment (including grading rubric) to your plan.

Houston, We Have a Word Problem Performance Task:
- Purpose: to assess students’ content knowledge based on being able to not only create problems but to also teach other people. The purpose is also to have students relate trig functions to real life situations.
- Students have the choice to work individually, in pairs, or (for the students in need of accommodation) in groups of three.
- Students are allowed to do research on the word problem. They will create a visual aid serving as a diagram for the word problem.
- The students will present the word problem and demonstrate their approach taken to solving it.
- The performance task will be introduced on Day 4. Students will have Day 5 in class and the weekend before the test to work on the project.
- Presentations will be on Days 7 and 8, after the unit test.

**Test/Quiz Item(s) and Other Traditional Assessments:** List the names of each test/quiz/homework/etc. here and attach a copy of each to your plan.

**Day 1:**
- “Unit Circle Opener” Google Poll Opener - [http://goo.gl/forms/H1R5gcYkHy](http://goo.gl/forms/H1R5gcYkHy)
- Calculate sinΘ, cosΘ, and tanΘ for Quadrants 2-4 (homework)

**Day 2:**
- “A Slice of Pi” Google Poll Homework - [http://goo.gl/forms/jo6ZxJ7VF0](http://goo.gl/forms/jo6ZxJ7VF0)

**Day 3:**
- “Periodic Graphs” Google Poll Opener - [http://goo.gl/forms/1IQZeySMNt](http://goo.gl/forms/1IQZeySMNt)

**Day 4:**
- “Trig Functions and Other Right Triangles” Google Poll Opener - [http://goo.gl/forms/qunp2nwnUh](http://goo.gl/forms/qunp2nwnUh)
  - Ladder word problem closer/homework

**Day 5:**
- Complete and turn in Ladder word problem homework

**Informal Check(s):** List ways you will check for understanding throughout your unit.

**Day 1:**
- *Now that our first special triangle has been positioned correctly, can anyone tell me what...*
these labeled numbers mean?

- The labeled numbers $\sqrt{3}$ and $\frac{1}{2}$ describe the distance travelled along the x-axis from the origin and the distance travelled along the y-axis from the x-axis, respectively, to form the top vertex of the triangle.

- Next, take the second yellow special triangle opposite the one we just positioned. Place the inner angle on top of the origin of the circle, as done with the other triangle. However, this time when we align the adjacent side of the triangle with the x-axis, in which quadrant will we be?
  - Quadrant II

- If the labeled numbers on the triangle are the x and y coordinates for the top vertex, why is it important that we know which quadrant we are in?
  - The sign of x and y values in coordinate pairs depend on which quadrant they are in.

- What does this mean, then, for our second yellow triangle? Will any of our signs change for the labeled values?
  - Yes, the x-value will become negative since we are in Quadrant II, or since we are moving left along the x-axis to reach that point.

- Why didn’t the y-value change?
  - In Quadrant II, we still had to move up to get to that point, so the y-value stayed positive.

- Let’s move on to the next type of triangle. Everyone take one blue triangle that has its specified angle on the left side. Place that angle over the origin of the circle. What is the coordinate pair for the top vertex of the triangle?
  - $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

- For the special right triangles in Quadrant II, which other quadrant will they be needed?
  - Quadrant IV

- Which other quadrant will the Quadrant I triangles be needed?
  - Quadrant III

Day 2:

- Does any of you know the symbol used when dealing with the decimal 3.14159? Do not say it aloud; rather, write it on your whiteboard so no other group hears your answer.
  - The students should be able to draw the $\pi$ symbol to answer the question.

- OK, good. Now let’s all type in our calculator $\frac{\pi}{4}$. What is the decimal value to three places for that expression?
  - 0.785

- Now let’s all type in our calculator $\frac{\pi}{2}$. What is the decimal value to three places for that expression?
  - 1.571

- Which is smaller, $\frac{\pi}{4}$ or $\frac{\pi}{2}$?
  - $\frac{\pi}{4}$

Day 3:

- Can one of you raise your hand and tell me your own definition of a radian?
  - Take students’ responses and praise them for knowing what a radian is. Try to call
on or prompt students who are more likely to have forgotten the lesson from the day before

- **Can anyone tell me which value from the coordinate points on the unit circle is sinθ?**
  - The y-value
- **Can anyone tell me which value from the coordinate points on the unit circle is cosθ?**
  - The x-value

Day 4:
- **What positional ratios are we given?**
  - Hypotenuse and adjacent
- **What trig function does this mean we should use to find θ?**
  - Tangent

Day 5: Study Guide and In-Class work on Project
Day 6: Test Day
Days 7 & 8: Project Presentations

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**Academic Prompt(s): List higher level thinking questions used throughout the unit.**

Day 1:
- **After you have labeled each of the trig function values for the points in Quadrant I, I want you to write down, on an exit slip, one thing you notice about sinθ and cosθ in correspondence with the x and y values we already had labeled.**
  - This is academic prompt for their discovery thought process, since this has not yet been explicitly stated in class during the unit.

Day 2:
- **What do you observe about the radian measure of the central angle?**
  - An answer that shows the students are thinking in the right direction would be, “The radian measure is equal to the radius.” Although this is correct, this will not be true when the radius equals something other than 1. The students should be able to make this discovery later in the worksheet. It is not necessary for the teacher to make this observation for them.
  - A more direct answer, and the one stated on the teacher version worksheet, would be, “The radian measure is 1 rad.”

- **What do you expect the arc length to be with the angle measure is 3 radians?**
  - Since the radian measure has been declared at 2 units, the answer should be, “The arc length is 6 units based on a radius of 2 units.”

Day 3:
- **Can anyone tell me their own definition of a periodic function?**
  - The actual definition of a periodic function is a function returning to the same value at regular intervals.
# Stage 3 – Daily Lesson Plans

(Stage 3 - attach lesson plans)

Make a calendar to outline the objectives taught each day, the activities/strategies used and the assessments used. Next, attach a separate lesson plan for each day of your unit using the format on the following page.

<table>
<thead>
<tr>
<th>STAGE 3: Daily Plans</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TSW</strong></td>
<td>Identify the four quadrants of the Cartesian plane. (DOK 1)</td>
<td>Tell the relationship between a central angle, the radius, and the arc length of a circle. (DOK 1)</td>
<td>Distinguish between periodic graphs and non-periodic graphs. (DOK 2)</td>
<td>Calculate the values of sine, cosine, and tangent for $x$, $\pi + x$, and $2\pi - x$ in terms of their values for $x$, where $x$ is any real number. (DOK 1)</td>
<td>Recall all material learned so far that will be included on the test. (DOK 1)</td>
</tr>
<tr>
<td></td>
<td>Distinguish in which quadrants $x$-values are positive or negative and $y$-values are positive and negative. (DOK 2)</td>
<td>Recognize that changing the radius of a circle does not affect the proportions in the circle. (DOK 1)</td>
<td>Interpret and categorize graphs into categories of periodic or not periodic. (DOK 2)</td>
<td>Relate the ratios $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, and $\tan\theta = \frac{y}{x}$ to any other right triangle with real numbers. (DOK 1)</td>
<td>Organize thoughts for completion of upcoming project. (DOK 2)</td>
</tr>
<tr>
<td></td>
<td>Construct a unit circle using special right triangles specific to the unit circle. (DOK 2)</td>
<td>Apply concepts to convert an angle measure from radians to degrees and vice versa. (DOK 4)</td>
<td>Calculate the decimal values for each radian measure on the unit circle. (DOK 1)</td>
<td>Interpret the ratios $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, and $\tan\theta = \frac{x}{y}$ to mean $\frac{\text{opposite}}{\text{hypotenuse}}$, $\frac{\text{adjacent}}{\text{hypotenuse}}$, and $\frac{\text{opposite}}{\text{adjacent}}$ for other right triangles not on the unit circle. (DOK 3)</td>
<td>Study Guide</td>
</tr>
<tr>
<td></td>
<td>Develop a logical argument explaining the coordinate points of the unit circle formed by the length of the legs of special right triangles. (DOK 3)</td>
<td>Create a proportion that can be used to convert the measure of an angle from degrees to radians and vice versa. (DOK 4)</td>
<td>“Periodic Graphs” Google Poll Opener</td>
<td>“Trig Functions and Other Right Triangles” Google Poll Opener</td>
<td>Work on projects</td>
</tr>
<tr>
<td></td>
<td>Label the $x$ and $y$ coordinates for the major points on the unit circle. (DOK 1)</td>
<td>Use appropriate tools strategically (DOK 1, Algebra CCSS Mathematical Practice).</td>
<td>Sin$\theta$ and Cos$\theta$ Handout</td>
<td>Rotating Triangles group activity</td>
<td>Complete and turn in Ladder word problem homework</td>
</tr>
<tr>
<td></td>
<td>Geometrically calculate the values for sine, cosine, and tangent for $\pi/3$, $\pi/4$ and $\pi/6$ using special</td>
<td>Look for and make use of structure (DOK 1, Algebra CCSS Mathematical</td>
<td>Drawing of sine and cosine graphs</td>
<td>Ladder word problem</td>
<td></td>
</tr>
<tr>
<td></td>
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</tr>
</tbody>
</table>
right triangles. (DOK 1)

Draw the conclusion that each coordinate point on the unit circle contains the sine and cosine for each angle in radian measure. (DOK 3)

"Unit Circle Opener" Google Poll Pre-Assessment

Interactive Unit Circle Handout Construction

Group work for finding $\sin \theta$, $\cos \theta$, and $\tan \theta$

Calculate $\sin \theta$, $\cos \theta$, and $\tan \theta$ for the remaining quadrants for homework

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<table>
<thead>
<tr>
<th>Practice.</th>
<th>TSW</th>
<th>TSW</th>
</tr>
</thead>
<tbody>
<tr>
<td>TI Inspire Radian Measure Worksheet Group Work</td>
<td>Create a word problem. (DOK 4)</td>
<td>Create a word problem. (DOK 4)</td>
</tr>
<tr>
<td>Parking Garage exit slip</td>
<td>Apply concepts of trigonometric functions and right triangles to real life situations. (DOK 4)</td>
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</tr>
<tr>
<td>&quot;A Slice of Pi&quot; Google Poll Homework</td>
<td>Show these ideas in as a presentation to the class. (DOK 2)</td>
<td>Show these ideas in as a presentation to the class. (DOK 2)</td>
</tr>
</tbody>
</table>

Present projects

Parking Garage exit slip

closer/homework

Introduction to Performance Task ("Houston, We Have a Word Problem")

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Parking Garage exit slip
Day 1:
Daily Lesson Plan

Day: (Day 1)
Objectives:

- **TSW**
  - Identify the four quadrants of the Cartesian plane. (DOK 1)
  - Distinguish in which quadrants x-values are positive or negative and y-values are positive and negative. (DOK 2)
  - Construct a unit circle using special right triangles specific to the unit circle. (DOK 2)
  - Develop a logical argument explaining the coordinate points of the unit circle formed by the length of the legs of special right triangles. (DOK 3)
  - Label the x and y coordinates for the major points on the unit circle. (DOK 1)
  - Geometrically calculate the values for sine, cosine, and tangent for π/3, π/4 and π/6 using special right triangles. (DOK 1)
  - Draw the conclusion that each coordinate point on the unit circle contains the sine and cosine for each angle in radian measure. (DOK 3)

Materials:

- Laptops
- “Unit Circle Opener” Google Poll - http://goo.gl/forms/H1R5gcykHy
- Yellow copy paper (Triangle placed first)
- Blue copy paper (Triangle placed second)
- Green copy paper (Triangle placed third)
- White copy paper
- Handout – “Interactive Unit Circle”
- Single hole punch
- Brads
- Scissors
- Pencils/pens for students to take notes on side of handout
- Completely assembled Interactive Unit Circle
- White board (for notes)
- Dry erase markers (for notes)
- Notebook paper (for calculations)

Opening (Set): 5 minutes

- *Hi, class! Today will begin our new unit on the unit circle! A little bit of arts and crafts will be involved here as we will be cutting out special right triangles to help us understand many of the connections that can be made in the unit circle.*
- *We will have a test on Monday, the sixth day of this unit. So keep that in mind as we go through the upcoming days of information.*
- *As I hand out the needed materials for today’s activity, everyone should follow the link to the class poll. This will hopefully give you a better introduction to today’s lesson and also let me see where we stand as a class in terms of prior knowledge of the subject.*
  - “Unit Circle Opener” Google Poll - http://goo.gl/forms/H1R5gcykHy
  - Answers to Poll will be analyzed as the answers come in. The students will be placed into heterogeneous groups based on whether or not a student answered the questions correctly.
• Let's go over the answers to the class poll.
  o The teacher should review with the class the correct answers to the poll so that they can efficiently use this information later.

Learning Tasks (Procedures): 40 minutes
I. Give instructions for assembling the Interactive Unit Circle.
   a. We will begin our unit circle activity by cutting out all of our special triangles. Do not cut out the large circles, only cut out the triangles.
   b. Use the hole punch to create a hole in the origin of your circle.
   c. Make sure you leave attached the circle at the vertex of each triangle in order to be able to attach them to the larger circle.
   d. Once all of our triangles are cut out and ready to be attached, use the hole punch to create a hole in the circle attached to each triangle.
   e. Each student should have two yellow triangles, two blue triangles, and two green triangles.
   f. Once our constructing activity is over, we will go over important parts of the unit circle. There will be room beside your unit circle to take notes and write down any important information we may need to refer back to later.
   g. I will be walking around to answer questions and to help you in this assembling process.
      i. Each student, although in groups, should now be cutting out his or her special triangles.

II. Explain the order in which to attach the special right triangles while explaining the labels on each triangle.
   i. Once all triangles have been cut out, the teacher will, triangle by triangle, demonstrate the order in which the triangles should be attached to the circle.
   ii. For those students who would like more assistance, a fully completed Interactive Unit Circle will available to pass around the classroom for reference.
   iii. While students are attaching each triangle one at a time together as a class, the teacher will explain why each triangle is placed in its particular position.
   b. Now that all of you have your special triangles cut out, we will go over the order in which to attach them to our circles. We will not attach the triangles until we have gone over each triangle as a class.
   c. First, take the yellow special triangle that is being projected to the right, with its specified angle at the left. Place the circle around the vertex at the origin of our unit circle, and align the x-axis with the side adjacent to that angle.
      i. Teacher should hold up completed Interactive Unit Circle as a reference.
   d. Now that our first special triangle has been positioned correctly, can anyone tell me what these labeled numbers mean?
      • The labeled numbers $\frac{\sqrt{3}}{2}$ and $\frac{1}{2}$ describe the distance travelled along the x-axis from the origin and the distance travelled along the y-axis from the x-axis, respectively, to form the top vertex of the triangle.
• They form the coordinate pair for the top vertex of the triangle.

e. Next, take the second yellow special triangle opposite the one we just positioned. Place the inner angle on top of the origin of the circle, as done with the other triangle. However, this time when we align the adjacent side of the triangle with the x-axis, in which quadrant will we be?
• Quadrant II

f. If the labeled numbers on the triangle are the x and y coordinates for the top vertex, why is it important that we know which quadrant we are in?
• The sign of x and y values in coordinate pairs depend on which quadrant they are in.

g. What does this mean, then, for our second yellow triangle? Will any of our signs change for the labeled values?
• Yes, the x-value will become negative since we are in Quadrant II, or since we are moving left along the x-axis to reach that point.

h. Why didn’t the y-value change?
• In Quadrant II, we still had to move up to get to that point, so the y-value stayed positive.

i. Let’s move on to the next type of triangle. Everyone take one blue triangle that has its specified angle on the left side. Place that angle over the origin of the circle. What is the coordinate pair for the top vertex of the triangle?
• \( \left( \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right) \)

III. Begin assembling the Interactive Unit Circle

a. At this point, we should all be able to see that the green triangle goes last, and this is just to keep all the labeled distances visible once our triangle is assembled. The order of assembly for each triangle does not make one more important than another.

b. Within your groups, begin layering the rest of the triangles in the manner we did as a whole class. Once all triangles are positioned correctly, have your group members check you, and then insert the brad into the origin of the circle and close it in the back.

c. It is important to note that \( \Theta \) is the angle measure projected by each special right triangle. On the outer ring of our Interactive Unit Circle are the measures of each \( \Theta \) in degrees. On the inner ring of Interactive Unit Circle are the measures of each \( \Theta \) in radians. We will learn more about radians another day, but for now just know that each radian measure contains \( \pi \), and is equal to its corresponding angle measure in degrees. For today, we will just be using the angle measure in degrees.

IV. In groups, and without the full direction of the teacher, label the coordinate pairs of the points on the unit circle using the special right triangles.

a. With your group members and using your special triangles, begin going around the entire circle and labeling the coordinate pairs for each major point.

b. Keep in mind what we discussed about quadrants and signs.

c. Also, remember that the brad in the center allows our special right triangles to rotate around the origin as needed. For the special right triangles in Quadrant II, which other quadrant will they be needed?
V. Take notes on the unit circle and trigonometric functions as they apply to the unit circle. Apply them to the unit circle to add more detail to the notes for a better understanding. This section relies on the use of grouping and clear verbal or modeled instructions for the students.

a. Using the space on the side of our Interactive Unit Circle, we will now write down a couple of important things we need to know and need to be able to relate back to the unit circle.

b. Before we begin taking notes, I need each student to take out his or her own notebook paper in order to perform necessary calculations later, so as to not clutter up our Interactive Unit Circle since we will be using it for other activities later in the unit.

c. The following are ratios of the measurements of the triangle containing Θ used to find Sine, Cosine, and Tangent of Θ.

   i. Write on the board:

   \[
   \sin(\Theta) = \frac{y}{r} \\
   \cos(\Theta) = \frac{x}{r} \\
   \tan(\Theta) = \frac{y}{x}
   \]

d. Using our Interactive Unit Circle's special right triangles and our ratios for these trig functions, we can find \(\sin\Theta\), \(\cos\Theta\), and \(\tan\Theta\) for any of the triangles.

e. Let's try to apply this concept to the unit circle by using one of our special right triangles.

f. Within your groups, choose a special right triangle for which you will be performing these calculations.

   i. Have students, in their groups, choose a special right triangle for which they will calculate \(\sin\Theta\), \(\cos\Theta\), and \(\tan\Theta\) using the notes just previously taken.

   g. Write these calculations on your separate sheet of paper for the time being, as we will be working in our groups to come up with the correct answers rather than relying on me at the board to work it out for you.

   h. Each group member will take one trig function for himself to calculate for \(\Theta\). Once every member of the group has completed his calculations, each member of the group will explain the process to the rest of the group of how he got his answer.

   i. Model the instructions quickly and efficiently, so that all groups understand and can start from there.
For instance, let's look at Group (1) with (Sally, Jake, and Robert). Sally will choose one trig function for herself, say \( \cos \theta \). This leaves Jake to choose, say, \( \tan \theta \), and Robert will find \( \sin \theta \).

Once Group (1) has completed all of their calculations, Sally will explain to Jake and Robert how she found \( \cos \theta \), Jake will explain to Sally and Robert how he found \( \tan \theta \), and Robert will explain to Sally and Jake how he found \( \sin \theta \).

It is important to understand that your group members are allowed to point out mistakes and help you come to the correct calculation if your final answer turns out to be wrong in your explanation to your group. This would be the appropriate time to ask for help, rather than relying on group members before trying on your own.

However, it is also important to understand that if your partner does ask for your help in the beginning of the process, try to lead them to the answer rather than simply doing it for them.

Teacher should walk around at this point to make sure, first, that students are not disabling the problem solving skills of their group members by simply doing the work for them and, second, that all groups are collaborating as opposed to leaving all the work to one person.

VI. **Have groups come to the board to show the class how they each calculated \( \sin \theta \), \( \cos \theta \), and \( \tan \theta \). Rotate \( \sin \theta \), \( \cos \theta \), and \( \tan \theta \) among group members once they come to the board.**

   a. At this point, we will have groups come to the board to show their work in finding the values of the trig functions for our special right triangles. I need three groups that each chose a different special right triangle than each other.

   b. When we come to the board, we will rotate trig functions among our group members. For instance, if you calculated for \( \cos \theta \), you will now use what you learned from your group member to calculate either \( \sin \theta \) or \( \tan \theta \).

   c. Which group who had the yellow special right triangle would like to volunteer?

   d. Which group who had the blue special right triangle would like to volunteer?

   e. Which group who had the green special right triangle would like to volunteer?

   f. Good, now these groups will come up one at a time and show their calculations. The rest of you will follow along with them and write down their calculations with them, just as if they were your teacher! Use your calculations sheet (separate sheet of paper) instead of your Interactive Unit Circle.

   g. Please feel free to ask any questions or express any concerns you have with the presenting groups’ calculations. We learn by making mistakes, so don’t feel afraid to make any or to accept help.

   i. The three groups who volunteered will come up one at a time and show their calculations for each trig function. The teacher should sit down and let them teach. The teacher should also allow time for the students to point out mistakes before interjecting. If the students do not notice a mistake quickly enough, it is important for the teacher to interject in a timely manner in order to prevent any confusion.
Closure: 5 minutes

- Today, we learned about the unit circle and how special right triangles relate to its concepts.
- We talked about the ratios with respect to the radius of a circle and the x and y coordinates of a point on that circle that can be used to solve for \( \sin \theta \), \( \cos \theta \), and \( \tan \theta \).
- For the end of class, I want each person to label \( \sin \theta \), \( \cos \theta \), and \( \tan \theta \) for each angle measure in the FIRST QUADRANT ONLY. This is the interval \( 0 \leq \theta \leq \frac{\pi}{2} \). You may label them anywhere on your Interactive Unit Circle paper.
- After you have labeled each of these, I want you to write down, on an exit slip, one thing you notice about \( \sin \theta \) and \( \cos \theta \) in correspondence with the x and y values we already had labeled.
  - Academic prompt for their discovery thought process, since this has not yet been explicitly stated in class during the unit.
- Also write down any questions or concerns you have about this lesson or the trig functions as they relate to the unit circle.
- For homework, everyone should calculate \( \sin \theta \), \( \cos \theta \), and \( \tan \theta \) for the remaining quadrants: Quadrant II, Quadrant III, and Quadrant IV. You will also need to find the values of these trig functions for the four major points on the circle: (0,1), (1,0), (-1,0), and (0, -1). Do this on a separate sheet of paper so that tomorrow we can check your work before labeling it on our Interactive Unit Circle.

Differentiated Instruction:

- **Enrichment:** The students who are making obvious advancements in the process ahead of regular time will be allowed to move forward onto the other quadrants to calculate the trig functions for those values of \( \theta \).

- **Intervention:** Students in need of intervention will be recognized after the data for the opener comes in from the google poll. They will be grouped with advanced learners in order to receive help during the class without causing the teacher to take time out of class instruction.

- **Accommodation:** Students in need of accommodation will be told that the x and y values are the \( \cos \theta \) and \( \sin \theta \), since their discovery skills may not be as advanced as those of the rest of the class.
Copy on Colc "A" paper.
Copy on Color
"b" paper.
Directions for the teacher:

Copy enough of the circles for the number of students you have in your classes. For the triangles, the sheet to copy on color "A" paper will make 6 sets per copy. So if I have 30 students, I only need to make 5 copies of this sheet. The sheet to copy on color "B" paper will make 3 sets per copy. The sheet to copy on color "C" paper will make 6 sets per copy.

Have the students cut out the triangles and assemble their Unit Circle Visual Aide. You will need to have a single holepunch as well as brads available for students to complete their visual aide. See the pictures on the next page for the final assembled Unit Circle. Students can now rotate the reference triangles to the appropriate quadrant in order to determine the coordinates of points on the Unit Circle.
Students could use this space to take notes...
or you could have them cut out circle and have no extra room for notes.
Unit Circle Opener

Check for Readiness

The following is an image of the unit circle. Before continuing, focus on the signs of each coordinate pair and the quadrant in which it lies. Consider what might make each value positive or negative.
Unit Circle Opener

* Required

For each of the following Quadrants on a Cartesian plane, tell the sign on any x-value and any y-value in the quadrant.

**Quadrant I**
- y is negative and x is negative
- x is positive and y is negative
- x is positive and y is positive
- y is positive and x is negative

« Back  Continue »

40% completed
Unit Circle Opener

* Required

**Quadrant II**

- y is positive and x is positive
- x is positive and y is negative
- x is negative and y is negative
- y is positive and x is negative

[Continue]
Unit Circle Opener

* Required

**Quadrant III**
- [ ] x is positive and y is negative
- [ ] x is positive and y is positive
- [ ] y is positive and x is negative
- [ ] y is negative and x is negative

< Back  Continue »

80% completed
Unit Circle Opener

* Required

**Quadrant IV**

- y is positive and x is negative
- x is positive and y is negative
- y is negative and x is negative
- x is positive and y is positive

[Submit]
Unit Circle Opener

Thank you for your response! Once everyone has submitted a response, we will form groups to start our lesson.

Submit another response

This form was created using Google Forms.
Create your own
Day 2:
Daily Lesson Plan

Day: (Day 2)
Objectives:
- TSW
  - Tell the relationship between a central angle, the radius, and the arc length of a circle. (DOK 1)
  - Recognize that changing the radius of a circle does not affect the proportions in the circle. (DOK 1)
  - Apply concepts to convert an angle measure from radians to degrees and vice versa. (DOK 4)
  - Create a proportion that can be used to convert the measure of an angle from degrees to radians and vice versa. (DOK 4)
  - Use appropriate tools strategically (DOK 1, Algebra CCSS Mathematical Practice).
  - Look for and make use of structure (DOK 1, Algebra CCSS Mathematical Practice).

Materials:
- Laptops
- TI Inspire software (installed before the class, preferably at beginning of school year)
- TI Inspire Radian Measure Worksheet – teacher version
- TI Inspire Radian Measure Worksheet – student version
- Radian Measure.tns file
- Apple TV
- Calculator
- Pencil/pen for filling out worksheet
- Car shaped cutouts on cardstock for closer
- “Parking Garage” pocket-board for closer
- “A Slice of Pi” Google Poll Homework Link - http://goo.gl/forms/jo6ZxJ7VFO

Opening (Set): 20 minutes
- Everyone pull out your Interactive Unit Circle we worked on yesterday and your calculations sheet. You were expected to have calculated \( \sin \Theta, \cos \Theta, \) and \( \tan \Theta \) for each angle measure in Quadrant II, III, and IV, including the values of the four major points on the circle.
- To begin class, we will make sure everyone’s calculations are correct, go over any mistakes you may have made while calculating, and label the correct values of the trig functions for each angle measure on our Interactive Unit Circle.
  - The teacher should walk around and make sure each student completed all of his/her calculations from the homework the night before.
  - The teacher will call on students to come to the board and work each of the calculations for the class to see. (Three trig functions for each remaining angle measurement, leading to a total of 40 short problems.)
    - It would be helpful for the teacher to call on students he/she noticed did something wrong when spot checking the homework for completion. This
will benefit other students who also made mistakes but do not want to speak up and ask questions.

- The students should be showing their work and justifying their answers using the ratios $\frac{y}{r}$, $\frac{x}{r}$, and $\frac{y}{x}$.
  - This especially goes for those students having a hard time grasping the concept. These ratios will need to be applied later to any other triangle using real numbers.
- The teacher should correct any mistakes made in the students’ calculations, and label his/her own unit circle along with them on the board so they understand visually what is expected of them on their own Interactive Unit Circle.

Learning Tasks (Procedures): 25-27 minutes

I. Introduce the TI Inspire Radian Measure Worksheet
   a. The teacher should already have his/her own computer pulled up with the Radian_Measure.tns file on the TI Nspire software.
      i. Screenshots of what the software should look like for both the students and the teacher:

Radian Measure

Drag the open circle to change the measure of the angle.

Use the slider to change the length of the radius.
\[ \theta = 2.1 \text{ rad} \]

\[ \text{arclength} = 2.1 \ u \]
b. The first page on the software (page 1.1) should be projected from the teacher’s computer onto the Apple TV while the teacher instructs the students to open the file.

c. Now that we are finished going over the homework, let’s discuss what we will be doing today. Today's lesson will be hands-on with our TI Nspire technology on our laptops.

d. Yesterday, we learned about the three trig functions sine, cosine, and tangent. We solved for \( \sin \theta, \cos \theta, \) and \( \tan \theta \) for the main angle measures on the unit circle.

e. We mentioned that the values on the inner circle of our Interactive Unit Circle were the angle measures in radians.

f. Our goal today is to develop a mathematical understanding of what a radian is. Once we have developed that understanding as a class, we will use that to convert from radians to degrees and from degrees back to radians.

II. Place the students in groups based on opener performance.

a. I will place you in groups as I hand out the worksheet.

   i. The teacher should now place the students in groups based on what understandings, or lack of understandings, were seen from the review of their homework.

   ii. Worksheets should be handed out during the grouping process.
iii. It should already be established for this worksheet that students in need of intervention will be automatically placed with students who are more advanced. This will be important for the task of the day, since it is mainly a discovery activity. Those who have a hard time visualizing structures and grasping abstract concepts will be placed with more advanced peers to help them better understand.

iv. The more advanced students who help slower progressing peers will also gain from the experience by reiterating their own understanding through teaching it to another person.

b. *As I am handing out the worksheets, follow the link on the class website to open the file in the TI software. I will also be handing out whiteboards to use during the activity.*

i. The link on the classroom website is the `Radian_Measure.tns` necessary for the students to complete the activity. This is the hands-on technology portion of the activity with which the worksheet follows along.

III. **Do number one together as a class. Use this time to explain how to use the software for the questions on the worksheet.**

a. The teacher should be at his/her desk using his/her computer and showing it on the Apple TV while all the students follow along on their own computers.

b. *We will do number one together as a class. Everyone follow along on your own computers.*

c. *As the instructions say, everyone should now be moved on to page 1.2 to start the activity.*

i. The teacher should also do this on his/her computer as it is projected at the front of the classroom to help guide students.

d. *Number one says to drag the open circle until the arc length and the radius are equal. You should all move your mouse over the open circle on the page until an open hand appears. When that hand appears, you can click on the circle and move it along the arc. This changes the measurement of the arc length.*

e. *Notice that the measurement for the arc length is declared at the bottom of the page, and the measurement for the radius is declared to the left of that.*

i. The teacher should hover the mouse over these two variables in order to point out to students where they should be looking on the page.

f. *As you can see by the radius measure, there are up and down arrows. These can be used to increase or decrease the measure of the radius. You will be instructed to use these arrows for other questions on the worksheet. But for number one we will keep the measure of the radius at 1 unit.*

g. *Now that we all know how to move around the circle and change the measurements of the radius and arc length, we can answer the question asked by number one. What do you observe about the radian measure of the central angle?*

- An answer that shows the students are thinking in the right direction would be, "The radian measure is equal to the radius."

Although this is correct, this will not be true when the radius equals something other than 1. The students should be able to make this discovery later in the worksheet. It is not necessary for the teacher to make this observation for them.
IV. Do number two together as a class. Have the students use one whiteboard per group to answer questions asked by the teacher.

a. Let's move on to number 2 together. The instructions say to drag the open circle farther along the arc. This lets you increase the measure of the arc length.

b. Hold up your whiteboards with your answers to the next questions as we think about them together.

c. What is the central angle measure when the length of the arc is twice the length of the radius?

- The students should hold up whiteboards to answer the question.
- The answer to this question is, “The radian measure is 2 rad.”

d. Use this information to prompt students to notice that the radian measure is not always just the length of the radius, as they may have incorrectly observed in number one.

e. Now, increase the measure of your radius to 2 units using the up arrow on the left side of the page.

f. What do you expect the arc length to be with the angle measure is 3 radians?

- Since the radian measure has been declared at 2 units, the answer should be, “The arc length is 6 units based on a radius of 2 units.”

g. At this point, if some students still seem to be struggling, direct them to answer this question with other radius measurements. This is for the teacher to decide whether or not some students need more practice to clearly see what is happening. Otherwise, they should get enough practice by working through the rest of the worksheet.

h. We have done numbers one and two together. Now in your groups, follow the instructions on the worksheet while using the TI software. I will be walking around to clarify anything and to monitor the progress of this activity. However, try to do as much as possible on your own and in your groups.

i. Once your group has completed number four, stop where you are and raise your hand. I will come to your group to review some necessary information needed for answering number 5.

V. Go to each group once they have reached number 5 and prompt them to realize the decimal value for $\pi$ is 3.14. Take this opportunity to discuss other decimal approximations such as $\frac{\pi}{4}$ and $\frac{\pi}{2}$.

a. This is the procedure for the teacher in prompting each group to realize the decimal value for $\pi$ is 3.14 and for discussing other decimal approximations using $\pi$.

b. Once a group raises their hands, the teacher should use the following prompts for each individual group:

c. I see that your group is now finished with number four and ready to move onto number five. Does any of you know the symbol used when dealing with the decimal 3.14159? Do not say it aloud; rather, write it on your whiteboard so no other group hears your answer.
• The students should be able to draw the π symbol to answer the question.

d. OK, good. Now let’s all type in our calculator \(\frac{\pi}{4}\) What is the decimal value to three places for that expression?
   • 0.785

e. Now let’s all type in our calculator \(\frac{\pi}{2}\) What is the decimal value to three places for that expression?
   • 1.571

f. Which is smaller, \(\frac{\pi}{4}\) or \(\frac{\pi}{2}\)?
   • \(\frac{\pi}{4}\)

g. Good job. Y’all are ready to move on and finish the rest of your worksheet!

h. I will be walking around to monitor your progress and to answer any questions.
   i. This is more of a discovery type of activity, so the direct instruction of the teacher should not be a main focus after this point.
   ii. Any students struggling with the concepts of today’s lesson should be able to ask their group members for clarification. If they are still struggling after this, the teacher should be available walking around the room to help.

VI. Use the Teacher Version of the Radian Measure Worksheet to monitor progress of students.
   a. The teacher version of the Radian Measure Worksheet provides a list of materials that have already been listed above in the materials section. It also provides tech tips in case the students have difficulties using the software.
   b. The answers to each question on the worksheet are given in the teacher version. This should be used by the teacher when he/she is walking around the classroom and monitoring student progress.

Closure: 3 minutes
• To wrap up class, everyone should turn in his or her own work sheet with your name written on it.
• I also want every individual person to turn in his/her own definition of a radian in his/her own words. Write them on our cars that I am handing out and park them in the garage on your way out the door. You may also write down any questions you have about today’s lesson or about anything we have learned in the unit so far.
• For homework, there will be a google poll posted to our class website. It should be completed before class tomorrow.
  o “A Slice of Pi” Google Poll Homework Link: http://goo.gl/forms/jo6ZxJ7VFO
• Just a reminder, your test will be next Monday!

Differentiated Instruction:
• Enrichment: Students who are more advanced will be placed with those students who are in need of more assistance. This will allow them to extend their own knowledge in a way that enables them to teach it to someone else. Teaching the information to someone else will allow the students to utilize their knowledge to the extent that they do not only understand what is happening, but they understand it enough to effectively communicate the information to someone else. All students should understand proper classroom
etiquette in terms of how much information is appropriate until someone is just giving out answers. This should have been established at the beginning of the school year.

- **Intervention**: Students who are in need of assistance understanding the concepts of today’s lesson will be placed with more advanced students who should be able to teach them the necessary information. The open ended teacher prompts using white boards should help these students along in the discovery process so that their train of thought is going in the right direction. The students should be showing their work and justifying their answers using the ratios \( \frac{y}{r} \), \( \frac{x}{r} \), and \( \frac{y}{x} \).

- **Accommodation**: Students in need of accommodation will be allowed to come up to the teacher’s desk and work through problems with the teacher. This alters the process of getting the answers where instead of having to heavily rely on more advanced students, slow his/her group down, or fall too far behind, the student will have more one-on-one guidance from the teacher.
Open the TI-Nspire document *Radian Measure.tns*.

In this activity, you will define a radian and discover how to convert from radians to degrees and vice versa.

---

Move to page 1.2.

1. Drag the open circle until the arc length and the radius are equal. What do you observe about the radian measure of the central angle?

2. Drag the open circle farther along the arc.
   a. What is the central angle measure when the length of the arc is twice the length of the radius?

   b. What do you expect the arc length to be when the angle measure is 3 radians? Explain your reasoning.

3. Click the slider to change the length of the radius. Are the observations you made in Questions 1 and 2 still true? Explain why or why not.

4. Define a radian.
5. Drag the open circle counterclockwise as far as possible.
   a. What is the approximate radian measure of the central angle?
   b. What symbol do we use for this approximation?
   c. What is the degree measure of the central angle?
   d. Write an equation to represent the relationship between the radian and degree measures of the central angle.

6. Drag the open circle until the central angle is a right angle.
   a. Write this approximation as an exact value.
   b. Write an equation to represent the relationship between the radian and degree measures of the right angle.

7. Click the slider to change the radius. Do the relationships you discovered in Questions 5 and 6 remain the same? Why or why not?

8. How could you determine the exact radian measure of a 45-degree angle?

9. How could you determine the degree measure of an angle that measures $\frac{7\pi}{12}$ radians?

10. Write a proportion that can be used for converting any radian measure to degree measure and vice versa.

11. Use the proportion from Question 10 to determine the radian measure of a 280-degree angle.
Math Objectives
- Students will describe the relationship between a central angle, the radius, and the arc length of a circle.
- Students will recognize that changing the radius of a circle does not affect the proportions in the circle.
- Students will convert an angle measure from radians to degrees and vice versa.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary
- central angle
- exact value
- radian
- right angle

About the Lesson
- This lesson involves exploring the relationship between the central angle, the arc, and the radius of a circle.
- As a result, students will
  - Use a slider to change the radius, and conclude that the relationships are preserved.
  - Form a definition of a radian.
  - Drag a point to change the measure of the central angle to discover the relationship between radian and degree measure.
  - Create a proportion that can be used to convert the measure of an angle from degrees to radians and vice versa.

**TI-Nspire™ Navigator™ System**
- Send out the Radian_Measure.tns file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials
- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:
- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.

Lesson Files:
- Student Activity
  - Radian_Measure_Student.pdf
  - Radian_Measure_Student.doc
- TI-Nspire document
  - Radian_Measure.tns
Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor (arrow) until it becomes a hand (＋) getting ready to grab the point. Also, be sure that the word point appears. Then select ‹ † to grab the point and close the hand (≡). When finished moving the point, select ″ to release the point. Select ‒ to hide the entry line if students accidentally select the chevron.

Tech Tip: Have the students tap the arrows on each slider to change the values of the slider.

Move to page 1.2.

1. Drag the open circle until the arc length and the radius are equal. What do you observe about the radian measure of the central angle?

Answer: The radian measure is 1 rad.

2. Drag the open circle farther along the arc.
   a. What is the central angle measure when the length of the arc is twice the length of the radius?

Answer: The radian measure is 2 rad.

b. What do you expect the arc length to be when the angle measure is 3 radians? Explain your reasoning.

Answer: The arc length is 6 units based on a radius of 2 units.

TI-Nspire Navigator Opportunity: Quick Poll
See Note 1 at the end of this lesson.

3. Select the slider to change the length of the radius. Are the observations you made in Questions 1 and 2 still true? Explain why or why not.

Answer: Yes, when the angle measure is 1 rad, the arc length and radius are still equal. The arc length remains proportional to the radius.
4. Define a radian.

**Answer:** A radian is the measure of a central angle that spans an arc whose length is equal to the length of the radius.

Move to page 1.3.

5. Drag the open circle counterclockwise as far as possible.
   a. What is the approximate radian measure of the central angle?

   **Answer:** The radian measure is approximately 3.14.

   b. What symbol do we use for this approximation?

   **Answer:** The angle measure is exactly \( \pi \).

   c. What is the degree measure of the central angle?

   **Answer:** The measure of a straight angle is 180 degrees.

   d. Write an equation to represent the relationship between the radian and degree measures of the central angle.

   **Answer:** \( \pi \) radians = 180 degrees

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**TI-Nspire Navigator Opportunity: Quick Poll**

See Note 2 at the end of this lesson.

6. Drag the open circle until the central angle is a right angle.
   a. Write this approximation as an exact value.

   **Answer:** The radian measure is approximately 1.57, which is exactly \( \frac{\pi}{2} \).

   b. Write an equation to represent the relationship between the radian and degree measures of the right angle.

   **Answer:** \( \frac{\pi}{2} \) radians = 90 degrees
7. Select the slider to change the radius. Do the relationships you discovered in Questions 5 and 6 remain the same? Why or why not?

**Answer:** Yes, the relationships remain the same because the measure of the central angle never changes when only the radius varies.

8. How could you determine the exact radian measure of a 45-degree angle?

**Sample Answers:** Answers will vary. Students might use the exact radian measure for 90 degrees and divide by 2 or use the exact radian measure for 180 degrees and divide by 4. They also might use a proportion. The exact radian measure of 45 degrees is \( \frac{\pi}{4} \).

9. How could you determine the degree measure of an angle that measures \( \frac{7\pi}{12} \) radians?

**Sample Answers:** Answers will vary. Students might just replace the \( \pi \) radians with 180 degrees and simplify, or they might use a proportion. The angle measure is 105 degrees.

10. Write a proportion that can be used for converting any radian measure to degree measure and vice versa.

**Answer:** \( \frac{\pi}{180} = \frac{radians}{degrees} \)

11. Use the proportion from Question 10 to determine the radian measure of a 280-degree angle.

**Answer:** \( \frac{\pi}{180} = \frac{x}{280} \) When solved, \( x = \frac{14\pi}{9} \)

**Wrap Up**
Upon completion of the discussion, the teacher should ensure that students are able to understand:
- The relationship between a central angle, the radius, and the arc length of a circle.
- Changing the radius of a circle does not affect the proportions in the circle.
- How to convert an angle measure from radians to degrees and vice versa.
Note 1
Question 2, *Quick Poll*
Send an open response Quick Poll to students asking for their predictions. Discuss as a class which of the predictions submitted they think will be the actual outcome and why.

Note 2
Question 5 part b, *Quick Poll*
Send an open response Quick Poll to see if students recognize the value 3.14159 as the approximation for $\pi$. Take this opportunity to discuss other decimal approximations, such as $\frac{\pi}{4}$ and $\frac{\pi}{2}$.
A Slice of Pi

Check for Understanding

* Required

Write your own definition of a radian. *

arc length = radius

1 radian

radius
Use a calculator to determine the decimal value for \( \pi/6 \) radians. Round to two decimal places.

0.52

This means that HOW MUCH of the radius has been wrapped around the circle to reach this point?

about half
A Slice of Pi

Use a calculator to determine the decimal value for π radians. Round to two decimal places. *

3.14

This means that how much of the radius has been wrapped around the circle to reach this point? *

a little over three radii
A Slice of Pi

* Required

Use a calculator to determine the decimal value for π/3 radians. Round to two decimal places.

1.05

This means that HOW MUCH of the radius has been wrapped around the circle to reach this point?

about one radius
A Slice of Pi

Your response has been recorded.

Submit another response

This form was created using Google Forms.
Create your own
Day 3:
Daily Lesson Plan

Day: (Day 3)
Objectives:
- TSW
  - Distinguish between periodic graphs and non-periodic graphs. (DOK 2)
  - Interpret and categorize graphs into categories of periodic or not periodic. (DOK 2)
  - Calculate the decimal values for each radian measure on the unit circle. (DOK 1)
  - Construct a graph of sine and cosine of each radian measure on the unit circle. (DOK 4)

Materials:
- “A Slice of Pi” Google Poll Homework Link - http://goo.gl/forms/jo6ZxJ7VFO
- Laptops
- Graded TI Nspire Radian Measure Worksheet
- “Periodic Graphs” Google Poll - http://goo.gl/forms/1IQZeySMNt
- Sine and Cosine Graph Tables handouts
- Calculators
- Poster-sized graphing paper
- Markers

Opening (Set): 10 minutes
- Yesterday, we learned what a radian is. Can one of you raise your hand and tell me your own definition of a radian?
  - Take students’ responses and praise them for knowing what a radian is. Try to call on or prompt students who are more likely to have forgotten the lesson from the day before.
- I will hand back your graded worksheets from yesterday.
  - The teacher should hand back the graded TI Nspire Radian Measure Worksheets to each student.
- Before we go over the new lesson, let’s review the homework poll from last night.
  - The teacher should go over all questions from “A Slice of Pi” homework poll to make sure the students know the correct answers for the upcoming test.
- Today, we will be talking about periodic graphs. The graphs of trig functions are periodic. Before I tell you the definition of a periodic function, we will take the opening Google Poll posted on the school website. Follow the link, and complete the poll on your own.
  - “Periodic Graphs” Google Poll - http://goo.gl/forms/1IQZeySMNt
  - Answers to Poll will be analyzed as the answers come in. This will allow the teacher to know which students are struggling with the interpretation of graphs and categorizing them into periodic or not periodic.
- Now, let’s review the poll to make sure everyone is on the same page.
  - The teacher should review the correct answers to the opening poll. This will help students who answered incorrectly to understand what is going on for this lesson.
  - The teacher should use this time to teach the students the definition of a periodic function.
- Can anyone tell me their own definition of a periodic function?
The actual definition of a periodic function is a function returning to the same value at regular intervals.

- Good! Now make sure we all keep that definition in mind for today's lesson. We will see through the graphs that sine and cosine are periodic functions.
- For the second portion of the learning task, students will be placed in homogenous groups based on who finishes the first part of the learning task at the same time.

Learning Tasks (Procedures): 37 minutes

I. Introduce the activity/worksheet for the day's lesson.
   a. Our activity today will be in two parts. The first part will be worked on your own. The second part will be in groups.
   b. As I mentioned earlier, the graphs of the trig functions are periodic. What we will be doing today is constructing graphs of sine and cosine.
   c. I have a worksheet for you that will help you along in the process of creating these graphs. The worksheet will be worked individually and with a calculator.
   d. It may be helpful for you to have out your Interactive Unit Circle, just in case you would like to refer back to it during today's activity.

II. Prompt students with questions to remind them of the information they may need to have in the front of their minds before beginning the activity.
   a. Before we get going into the activity, let's make sure we have all the necessary tools.
   b. Can anyone tell me which value from the coordinate points on the unit circle is \( \sin \Theta \)?
      i. The y-value
   c. Can anyone tell me which value from the coordinate points on the unit circle is \( \cos \Theta \)?
      i. The x-value
   d. The teacher should now pass out the worksheet.

III. Give instructions to begin the worksheet.
   a. What I want you to do for each table of points is to label the angle measures in radians, starting at 0, down the first column. Notice that the first two have been done for you.
   b. For the second column, label the \( \sin \Theta \) or \( \cos \Theta \), (whichever is labeled on the table). As we have established, these would be either the x- or y-value from each coordinate point of the terminal side of \( \Theta \).
   c. For the third column, use your calculator to find the decimal value of the \( \sin \Theta \) or \( \cos \Theta \) (whichever is labeled on the table).
   d. As you complete them on your own, you will be grouped with others who finish at the same time as you. You will then draw the graphs of sine and cosine together.
   e. Feel free to raise your hand if you have any questions about calculator input or what is expected of you. I will be walking around to check on everyone's progress.
      i. The teacher should walk around the room to be sure they are on task and using their calculators correctly.
IV. Use the table values to construct the graphs of sine and cosine.
   a. Now that we have all finished filling out our tables, we will get into groups to construct the graphs of sine and cosine.
      i. The teacher should place the students into the homogeneous groups whenever a group of students finish the worksheet at the same time.
      ii. The teacher should also hand out one sheet of poster-sized graphing paper and markers to each group. Both graphs will be drawn on one sheet of paper.
   b. Each group only has one sheet of graphing paper. You will need to draw BOTH graphs on this one sheet of paper. That means you will have two coordinate planes. Make sure to designate the top half of your paper for the graph of sine, and the bottom half of your paper for the graph of cosine, or vice versa.
   c. The values in the first column of your tables will be the values on the x-axis.
   d. The values in the third column of your tables will be the values on the y-axis. Both of these have been specified at the top of the tables.
   e. Make sure to have every person in your group plot some of the points so that everyone gets to participate.
   f. When all the graphs are completed, we will hang our works of art out in the hall for everyone who walks by to see what the graphs of sine and cosine look like!

V. Give the groups who finish early an enrichment exercise.
   a. Hopefully, the most advanced groups will finish their graphs early. These groups will be given the assignment to look up the definition of a function’s period, and label the specific period of sine and cosine on their graphs.

VI. Instructions for groups working at a normal pace:
   a. Groups working at a normal pace should be given the definition of a function’s period. They should then find the period for the graphs of sine and cosine and label them on their graphs.

VII. Work one-on-one with students in need of accommodation.
   a. The teacher should work one-on-one with students in need of accommodation. They will work together through the entire handout, and will not be expected to find the period of each graph on their own.
   b. The period of the sine and cosine graphs is $2\pi$.

Closure: 3 minutes
- On the back of your graph, write the names of all your group members.
- Today, we looked at a few examples of periodic graphs and non-periodic graphs. We also constructed our own graphs of sine and cosine.
- Now that we are all finished plotting our points and constructing our graphs, we can all go out in the hall and hang the finished products for everyone to see!
- Just a reminder, your test will be next week on Monday!

Differentiated Instruction:
- Enrichment: Since the activity for actually drawing the graphs is group work, the groups will be formed based on who finishes the handout at the same time. This will most likely place the advanced students together, the regularly performing students together, and the students in need of more help together. The advanced students will most likely finish
drawing their graphs early. At this point, they will be instructed to label the period of each function. They will not be told what a period is, they can look this up on their laptops if they need to. This will be an opportunity for them to teach themselves.

- **Intervention:** The students who take a bit more time to fill in the handout will be placed in groups together to draw the graphs of sine and cosine. This will cause the groups to, hopefully, have all the same questions/concerns about the activity. This way, no one person is doing all the work. These groups, if they finish in a timely manner, will be told the definition of a period. They will be required to label the periods of each function on their graph.

- **Accommodation:** Those students in need of accommodations will all be placed in a group together. If there is just one student in need of accommodation, he/she will work on a graph with the teacher. The teacher will explain one-on-one what a period is, and will help that student find the period of each graph. This changes the process of what is to be learned, so that the student can have one-on-one help from the teacher with less expectations of discovery.
NAME: __________

**Graph of Sine**

<table>
<thead>
<tr>
<th>Radian Measure (x-coordinates)</th>
<th>sinθ</th>
<th>Decimal Value of sinθ (y-coordinates)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\frac{\pi}{6}$</td>
<td>$\frac{1}{2}$</td>
<td>0.5</td>
</tr>
<tr>
<td>Radian Measure (x-coordinates)</td>
<td>cos$\theta$</td>
<td>Decimal Value of cos$\theta$ (y-coordinates)</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>------------</td>
<td>--------------------------------------------</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\frac{\pi}{6}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>0.866</td>
</tr>
</tbody>
</table>
Periodic Graphs

Check for Readiness

* Required

In your own words, what does it mean for something to be periodic? *

Give an example of something that is periodic. *

Mrs. Hancock's example would be the school year that begins in August and ends in May, only to repeat that next August.

This is a required question
Periodic Graphs

* Required

Definition of a Periodic Graph
a function that repeats its values in regular intervals or periods

Using the definition of a periodic graph, tell whether or not the above graph is periodic. *

- Yes, periodic
- No, not periodic

33% completed
Periodic Graphs

* Required

**Definition of a Periodic Graph**

A function that repeats its values in regular intervals or periods

![Graph Image]

Using the definition of a periodic graph, tell whether or not the above graph is periodic. *

- Yes, periodic
- No, not periodic
Periodic Graphs

* Required

Definition of a Periodic Graph
a function that repeats its values in regular intervals or periods

Using the definition of a periodic graph, tell whether or not the above graph is periodic. *

- Yes, periodic
- No, not periodic
Periodic Graphs

* Required

Definition of a Periodic Graph
A function that repeats its values in regular intervals or periods

Using the definition of a periodic graph, tell whether or not the above graph is periodic. *

- Yes, periodic
- No, not periodic
Periodic Graphs

* Required

Definition of a Periodic Graph
a function that repeats its values in regular intervals or periods

Using the definition of a periodic graph, tell whether or not the above graph is periodic.*

- Yes, periodic
- No, not periodic

< Back  Submit

Never submit passwords through Google Forms.

100%: You made it.
Periodic Graphs

Thank you for your response! We will use the information from this poll in our lesson today.

Submit another response

This form was created using Google Forms.
Create your own
Day 4:
Day: (Day 4)

Objectives:
- TSW
  - Calculate the values of sine, cosine, and tangent for $x$, $\pi + x$, and $2\pi - x$ in terms of their values for $x$, where $x$ is any real number. (DOK 1)
  - Relate the ratios $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, and $\tan\theta = \frac{y}{x}$ to any other right triangle with real numbers. (DOK 3)
  - Interpret the ratios $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, and $\tan\theta = \frac{y}{x}$ to mean $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, and $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ for other right triangles not on the unit circle. (DOK 3)

Materials:
- Laptops
- “Trig Functions and Other Right Triangles” Google Poll Opener - http://goo.gl/forms/qunp2nwnUh
- Triangle cut-outs
- Whiteboard
- Dry-erase markers
- Individual white boards for closing word problem
- Project rubric

Opening (Set): 5 minutes
- Before we begin class, I want to remind everyone that we have a test on Monday. Things that we learn in class today will be included on the test. So, pay attention. I know all of you will do great!
- Today, we will be jumping back a couple days to apply some concepts we learned the first day of the unit.
- First, I want you all to complete the opener Google poll by following the link posted on the class website.
  - “Trig Functions and Other Right Triangles” Google Poll Opener - http://goo.gl/forms/qunp2nwnUh
- Raise your hand if you have any questions. Don’t worry if you are a little confused at first. We will put these concepts into practice for class today.
- The teacher should review the correct answers to the poll.
- Now that we are all done with our poll, let’s make sure we all understand what is going on.
- The teacher should open the poll and explain the correspondence between the coordinate ratios and the positional ratios of the sides of the right triangles.

Learning Tasks (Procedures): 30 minutes
I. Group students based on their answers to the Google poll opener.
   a. There will be five groups of five for this activity. Advanced students will be placed into heterogeneous groups with lower performing students. These groups
will be determined by the Google poll opener. The teacher should analyze the data as it is received during the opener.

b. Now that we are all done with our opening poll, I will place you into groups for today’s activity.

II. Explain the day’s class activity.

a. Our activity today will require collaboration in groups to calculate \( \sin \theta \), \( \cos \theta \), and \( \tan \theta \) for triangles with measurements of real numbers.

b. I have five triangle cut-outs that we will be passing around among groups.

c. I will pass one triangle to each group. You will have 2 minutes to calculate all three trig functions for each triangle. I will time you. Once those 2 minutes are up, we will pass the triangles to the right to the next group.

d. There is no need for calculators, as I only want the exact answer to the trig functions for \( \theta \).

e. Write your group’s answers down on a separate sheet of paper. Go ahead and designate one person in your group to be the record keeper so things move smoothly.

f. Label each calculation as “T1” for triangle one, “T2” for triangle 2, and so forth.

g. It is important to understand that even if you have not completed the calculations for each trig function for your triangle, you still must pass your triangle when the time is up.

h. Once every triangle has been used by every group, each group will come to the front of the class and present their triangle and teach the calculations to the rest of the class.

III. Begin the activity.

a. The teacher should now pass out one triangle to each group. Lay the triangle face down.

b. Keep your triangles face down until I start the 2 minute timer. Once the timer is started, your group may start calculating.

c. When the timer goes off, quickly rotate your triangles.

d. When I say “Go!” you may turn over your triangles. Go!
   i. The students should be moving at a fast pace in order to calculate before time runs out. This should get them motivated. They should also be motivated to get the calculations correct since they will have to present it to the rest of the class.

e. The teacher should be starting and stopping the 2 minute timer five times. This should take about ten minutes.

IV. Have the students come to the front of the class and present their calculations.

a. At this point, the students should have found the exact values for the three trig functions for five triangles.

b. Y’all did such a great job keeping up with the timer! Now, one group at a time, come to the board and show us the triangle you are left with and show us how you got \( \sin \theta \), \( \cos \theta \), and \( \tan \theta \) for that triangle.

c. I want every person in each group to say something at the front of the class, even if it is just an observation you made during the process.
   i. This should take about three minutes per group, or fifteen minutes.
d. Use this opportunity to correct any mistakes made by the students.

Closure: 10 minutes

I. Have the class set up a word problem. Have them complete it for homework.
   • For the end of class, we will complete a word problem using the positional ratios we
     learned and used today.
   • I will also be introducing a project I want you to start working on that will be due
     Tuesday and Wednesday. I will hand out and explain the rubric after we complete this
     word problem.
   • Word problem: The teacher should write the following word problem on the board.
     The students may discuss and answer the word problem within their groups:
     ▪ A 10 ft. ladder is propped against a brick building. The base of the ladder is 7
       ft. away from the building. What is the angle between the ladder and the
       ground?
   • I want everyone individually to draw a diagram of this on your whiteboards.
   • What positional ratios are we given?
     ▪ Hypotenuse and adjacent
   • What trig function does this mean we should use to find Θ?
     ▪ Tangent
   • Good job. Now, set up an equation using tanΘ and the measurements for the
     hypotenuse and adjacent side to solve for Θ.
   • Now that you know the steps to find the answer to this problem, take this home and do
     it for homework tonight. It will be due at the end of class tomorrow.

II. Introduce the rubric for the upcoming project.
   • For our upcoming project, you will be creating your own word problem like the one on
     the board.
   • You may either work individually or get in pairs for this project.
     o Students in need of accommodation may join another pair, making it a group of
     three.
   • Do not use the ladder problem that we have already discussed and that you already have
     as a homework assignment.
   • I want you all to be creative with this.
   • Once you have a word problem, create a visual representation of the problem. This can
     be done on poster-board, created using 3D props, or created with computer software. I
     also want the work for this problem and the correct answer.
   • We will present these next week starting Tuesday. You will have to teach the class how to
     work your problem, and show us your visual aid.
   • I will now hand out the rubric for the project. After you have received your rubric, you
     are free to pack up.
   • Tomorrow will be a review day for your test on Monday. You will be working alone or in
     groups on a study guide I will give you. You also have the choice to go ahead and work
     on you projects in class. If you do choose to work on your project, bring the necessary
     materials to class.
• You are also allowed to bring tonight’s homework to work on in class tomorrow. You may work with a chosen partner for this assignment. However, regardless of when you decide to work on it, you must hand it in by the end of class tomorrow.
  o The homework should not be taken for an accuracy grade. It is simply to see how the students are able to process through a word problem like this one. Since this is the first time they have seen anything like it in the unit, it is more of a transition in to what will be taught after the test.
  o The teacher should now pass out the project rubric.

Differentiated Instruction:
• Enrichment: This activity is geared more towards the students who are more advanced in that it is very fast paced. The advanced students will also have a chance to teach these concepts to lower students.

• Intervention: Lower performing students will be placed with more advanced students during the fast-paced activity so that no one is necessarily left behind.

• Accommodation: Those students in need of accommodations will also be placed in groups with advanced and regularly performing students. These students will not be required to teach the calculations; instead, they can just tell observations they made from the process.
  o For the project, students in need of accommodation will be allowed to join another pair, creating a group of three.
Houston, We Have a Word Problem!

You will be creating your own word problem involving real life situations, trig functions, and angle measures.

Your project should include the following components:

_____/3 Complete word problem setting up a question. Must involve a real life scenario

_____/3 Visual aid serving as a diagram (media type for the visual aid is completely up to student

_____/3 Correct answer to the question being asked by the word problem

_____/3 Mathematical work done to reach the final answer

You will be graded on each of these things for a total of **12 points**.

The attached rubric will be the guidelines for how I will grade your presentation. Each section will be worth 3 points for a second total of **18 points**.

Your entire project grade will be out of a total of **30 points**.

Write your name on the rubric and turn it in to me at the time of your presentation.
# Houston, We Have a Word Problem!

**PRESENTATION RUBRIC for PBL**
(for grades 9-12; Common Core ELA aligned)

<table>
<thead>
<tr>
<th>Explanation of Ideas &amp; Information</th>
<th>Below Standard (1 point)</th>
<th>Approaching Standard (2 points)</th>
<th>At Standard (3 points)</th>
<th>Above Standard</th>
</tr>
</thead>
</table>
| **Explanation of Ideas & Information** | - does not present information, arguments, ideas, or findings clearly, concisely, and logically; argument lacks supporting evidence; audience cannot follow the line of reasoning  
- selects information, develops ideas and uses a style inappropriate to the purpose, task, and audience (may be too much or too little information, or the wrong approach)  
- does not address alternative or opposing perspectives | - presents information, findings, arguments and supporting evidence in a way that is not always clear, concise, and logical; line of reasoning is sometimes hard to follow  
- attempts to select information, develop ideas and use a style appropriate to the purpose, task, and audience but does not fully succeed  
- attempts to address alternative or opposing perspectives, but not clearly or completely | - presents information, findings, arguments and supporting evidence clearly, concisely, and logically; audience can easily follow the line of reasoning (CC 9-12.SL.4)  
- selects information, develops ideas and uses a style appropriate to the purpose, task, and audience (CC 9-12.SL.4)  
- clearly and completely addresses alternative or opposing perspectives (CC 11-12.SL.4) | |

<table>
<thead>
<tr>
<th>Organization</th>
<th>Below Standard (1 point)</th>
<th>Approaching Standard (2 points)</th>
<th>At Standard (3 points)</th>
<th>Above Standard</th>
</tr>
</thead>
</table>
| **Organization** | - does not meet requirements for what should be included in the presentation  
- does not have an introduction and/or conclusion  
- uses time poorly; the whole presentation, or a part of it, is too short or too long | - meets most requirements for what should be included in the presentation  
- has an introduction and conclusion, but they are not clear or interesting  
- generally times presentation well, but may spend too much or too little time on a topic, a/v aid, or idea | - meets all requirements for what should be included in the presentation  
- has a clear and interesting introduction and conclusion  
- organizes time well; no part of the presentation is too short or too long | |

<table>
<thead>
<tr>
<th>Eyes &amp; Body</th>
<th>Below Standard (1 point)</th>
<th>Approaching Standard (2 points)</th>
<th>At Standard (3 points)</th>
<th>Above Standard</th>
</tr>
</thead>
</table>
| **Eyes & Body** | - does not look at audience; reads notes or slides  
- does not use gestures or movements  
- lacks poise and confidence (fidgets, slouches, appears nervous)  
- wears clothing inappropriate for the occasion | - makes infrequent eye contact; reads notes or slides most of the time  
- uses a few gestures or movements but they do not look natural  
- shows some poise and confidence, (only a little fidgeting or nervous movement)  
- makes some attempt to wear clothing appropriate for the occasion | - keeps eye contact with audience most of the time; only glances at notes or slides  
- uses natural gestures and movements  
- looks poised and confident  
- wears clothing appropriate for the occasion | |
<table>
<thead>
<tr>
<th></th>
<th>Below Standard</th>
<th>Approaching Standard</th>
<th>At Standard</th>
<th>Above Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Voice</strong></td>
<td>• mumbles or speaks too quickly or slowly</td>
<td>• speaks clearly most of the time</td>
<td>• speaks clearly; not too quickly or slowly</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• speaks too softly to be understood</td>
<td>• speaks loudly enough for the audience to hear most of the time, but may speak in a</td>
<td>• speaks loudly enough for everyone to hear; changes tone and pace to maintain interest</td>
<td></td>
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<tr>
<td></td>
<td>• frequently uses “filler” words (“uh, um, so, and, like, etc.”)</td>
<td>• occasionally uses filler words</td>
<td>• rarely uses filler words</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• does not adapt speech for the context and task</td>
<td>• attempts to adapt speech for the context and task but is unsuccessful or inconsistent</td>
<td>• adapts speech for the context and task, demonstrating command of formal English when</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>appropriate (CC 9-12.SL.6)</td>
<td></td>
</tr>
<tr>
<td><strong>Presentation Aids</strong></td>
<td>• does not use audio/visual aids or media</td>
<td>• uses audio/visual aids or media, but they may sometimes distract from or not add</td>
<td>• uses well-produced audio/visual aids or media to enhance understanding of findings, reasoning,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• attempts to use one or a few audio/visual aids or media, but they do not add</td>
<td>to the presentation</td>
<td>and evidence, and to add interest (CC 9-12.SL.5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>to or may distract from the presentation</td>
<td>• sometimes has trouble bringing audio/visual aids or media smoothly into the presentation</td>
<td>• smoothly brings audio/visual aids or media into the presentation</td>
<td></td>
</tr>
<tr>
<td><strong>Response to Audience</strong></td>
<td>• does not address audience questions (goes off topic or misunderstands</td>
<td>• answers audience questions, but not always clearly or completely</td>
<td>• answers audience questions clearly and completely</td>
<td></td>
</tr>
<tr>
<td></td>
<td>without seeking clarification)</td>
<td></td>
<td>• seeks clarification, admits “I don’t know” or explains how the answer might be found when</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>unable to answer a question</td>
<td></td>
</tr>
</tbody>
</table>
Trig Functions and Other Right Triangles

Mini Lesson

Required

Right Triangle with Trig Ratios
The above triangle has the coordinate plane variables labeled. \( X \) is the distance from the origin along the \( x \)-axis, \( y \) is the distance along the \( y \)-axis, and \( r \) is the length of the terminal side of \( \theta \). Give the ratio for \( \sin \theta \). *

- \( x/r \)
- \( y/r \)
- \( y/x \)

Give the ratio for \( \cos \theta \).*

- \( x/r \)
- \( y/r \)
- \( y/x \)

Give the ratio for \( \tan \theta \).*

- \( x/r \)
- \( y/r \)
- \( y/x \)
Trig Functions and Other Right Triangles

* Required

Right Triangle with Position Ratios
The above triangle has sides labeled in terms of their position to $O$. Using what you know about the trig function ratios, give the ratios in terms of the labeled positions for $\sin O$.*

\[
\text{opposite/hypotenuse}
\]

Using what you know about the trig function ratios, give the ratios in terms of the labeled positions for $\cos O$.*

\[
\text{adjacent/hypotenuse}
\]

Using what you know about the trig function ratios, give the ratios in terms of the labeled positions for $\tan O$.*

\[
\text{opposite/adjacent}
\]
Trig Functions and Other Right Triangles

* Required

Position Ratios
\[ \sin \theta = \text{opposite}/\text{hypotenuse} \]
\[ \cos \theta = \text{adjacent}/\text{hypotenuse} \]
\[ \tan \theta = \text{opposite}/\text{adjacent} \]
Trig Functions and Other Right Triangles
Thank you for your hard work! This activity is an introduction to what we will be doing in class today.
Day 5:
Daily Lesson Plan

Day: (Day 5)
Objectives:
- **TSW:**
  - Recall all material learned so far that will be included on the test. (DOK 1)
  - Organize thoughts for completion of upcoming project. (DOK 2)

Materials:
- Test Study Guide
- Pencils/pens
- Laptops for researching/working on project
- Project materials (if the students chose to bring them)
- Previous day’s homework assignment (ladder word problem written on board)

Opening (Set): 2 minutes
- *Hi, class! For any of you who are already done with your homework assignment from last night, please turn it in now.*
  - The teacher should take up the homework from students who have completed it.
- *Today is a review day! I will be handing out study guides to everyone. Those who choose to work in pairs or groups of three on the study guide may do so.*
- *Those who chose to bring their project materials to work on in class may also do so.*
- *I will be completely available to all of you for any questions you may have on the study guide or your project. I will also be available to answer any questions you may have for the homework due by the end of class today.*
- *Although I will be coming around to answer any questions you may have, and what I am about to say goes especially for the study guide, I want you to be able to do these things on your own since you will have to do them on your own for the test. Therefore, don’t expect all the answers to be given to you.*
  - The teacher should hand out a study guide to each student.

Learning Tasks (Procedures): 46 minutes
I. Students start on the study guide, project, or continue with the previous night’s homework.
   a. *You may now start on whichever activity you chose to do today. I do, however, highly recommend you work on the study guide since your test is Monday.*

II. Explain when the homework assignment will be discussed as a class. Allow the students who need more help with the concept of the word problem to come to the teacher’s desk for individual or small group attention.
   a. *We will not be going over the correct answers to this homework question until after we present our projects. If you need a little more individual help on it, come to my desk right now and we can go over it together.*

Closure: 2 minutes
- *I hope all of you are ready for the test on Monday!*
- *If anyone has any questions before we leave class, please ask them now! Have a great day!*

Differentiated Instruction:

- **Enrichment:** Since all students have a choice on what they work on in class today, the more advanced students will be able to get to their project if they feel as though they understand everything on the study guide.

- **Intervention:** The teacher will be available to walk around and answer any questions students have. Those students in need of intervention will receive more guided instruction on their study guides than other students.

- **Accommodation:** Those students in need of accommodations will be allowed to come to the teacher's desk for individual or small group attention on the study guide or homework assignment. These students will be given more hints on the homework assignment.
Unit Circle, Trigonometric Functions, and Periodicity
Study Guide

Part 1: You are allowed to use a calculator. You will not be able to use the unit circle.
Part 2: You will not be allowed to use a calculator. You will be given a unit circle.

- Know the definition of a radian
- Know the proportions used for converting from radians to degrees
- Know the proportions used for converting from degrees to radians
  - For practice converting, use these proportions to go back and forth from radians to degrees for all the angle measurements on the unit circle.

- Know the definition of the period of a function.
- Know the period of the graph of sine
- Know the period of the graph of cosine
- Know the difference between a periodic graph and a non-periodic graph

- Know how to calculate \( \sin \theta \), \( \cos \theta \), and \( \tan \theta \) of any given right triangle with measurements of real numbers (positional trig ratios)

- Know the trigonometric ratios for \( \sin \theta \), \( \cos \theta \), and \( \tan \theta \)

- Be able to use these to find \( \sin \theta \), \( \cos \theta \), and \( \tan \theta \) for any given radian or degree measure on the unit circle.

You should be able to find these concepts throughout all the work we have done so far. This includes Google polls, classroom worksheets and classroom activities.
Day 6:
Daily Lesson Plan

Day: (Day 6)

Objectives:
- TSW:
  - Recall definitions. (DOK 1)
  - Use concepts from unit to answer test questions for “Unit Circle, Trigonometric Functions, and Periodicity” Test. (DOK 1)

Materials:
- “Unit Circle, Trigonometric Functions, and Periodicity” Part 1
- “Unit Circle, Trigonometric Functions, and Periodicity” Part 2
- Pencils/pens to take test
- Calculator
- Test answer key

Opening (Set): 2 minutes
- So far we have done a great job of staying on task and learning the material. I appreciate every bit of y’all’s time and effort put into completing this unit!
- Please put away all notes, study guides, or anything else you have on your desks besides a pencil and a calculator.
- For the first part of the test, you will be able to use a calculator. Once you have completed Part 1 of the test, turn it in to me and you will receive Part 2. At that point, you must put away your calculator.
- If you have any questions during the test, raise your hand and I will come to you.
- You may start as soon as I hand you the test. Good luck, everyone!
- The teacher should start handing out Part 1 of the test to all students.

Learning Tasks (Procedures): 45 minutes

I. Hand out the tests.
   a. The teacher should hand out a copy of the test to each student.
   b. The teacher should be available to go to a student’s to answer any questions they may have.
   c. Once a student finishes Part 1, the teacher should give them Part 2.
   d. They cannot use a calculator on Part 2.

Closure: 2 to 3 minutes
- It is now almost time to go. If you have not finished your test, please bring them up to me to turn in.
- Remember that we have a project due tomorrow for presenting. We will present in random order. I will draw your names from a jar of Popsicle sticks. That means that everyone must come prepared to present, even if we don’t get to you tomorrow.
- Have a great day!

Differentiated Instruction:
- Enrichment: The students who are more advanced will take the given test.
• **Intervention:** Students in need of intervention will be allowed to come back at another time of the day or week to complete the test if they have not finished by the time class is over.

• **Accommodation:** Students in need of accommodation will also be allowed to come back at another time of the day or week to complete the test if they have not finished by the time class is over.
  - They will also be given definitions, ratios, and proportions asked for on the test. This will be a separate version of the test.
  - Instead of losing credit per question, these students will lose credit per concept section.
Unit Circle, Trigonometric Functions, and Periodicity
Part 1
The first part of the test will be completed without a unit circle. Once you have turned in this section, you will receive the second part of the test with a unit circle. You may use a calculator for this portion of the test.

• Give the definition of a radian.

• Write the proportion used for changing from radians to degrees

• Write the proportion used for changing from degrees to radians

Convert each of the following radian measures to degrees.

1. \(\frac{\pi}{2}\)  
2. \(\frac{6\pi}{7}\)  
3. \(\frac{11\pi}{6}\)

4. \(\frac{3\pi}{2}\)  
5. \(\pi\)  
6. \(\frac{\pi}{9}\)

7. \(2\pi\)  
8. \(\frac{5\pi}{8}\)  
9. \(\frac{8\pi}{11}\)

Convert each of the following degree measures to radians.

1. 55°  
2. 75°  
3. 100°

4. 410°  
5. 290°  
6. 30°

7. 200°  
8. 310°  
9. 97°
Give the definition of the period of a function.

What is the period of sine?

What is the period of cosine?

Tell whether or not each of the following graphs are periodic.

A. Periodic
B. Not Periodic

A. Periodic
B. Not periodic
A. Periodic
B. Not periodic

A. Periodic
B. Not Periodic
Unit Circle, Trigonometric Functions, and Periodicity
Part 2
You may NOT use a calculator for this section of the test.
Use the given unit circle to answer the following section’s questions.
Give the trigonometric ratio for each of the following:

\[
\sin \theta = \quad \cos \theta = \quad \tan \theta =
\]

Calculate \( \sin \theta \), \( \cos \theta \), and \( \tan \theta \) for each of the following radian or degree measures.

1. \( \theta = 225^\circ \)

2. \( \theta = \frac{5\pi}{3} \)

3. \( \theta = 90^\circ \)

4. \( \theta = \frac{\pi}{6} \)

5. \( \theta = 270^\circ \)

6. \( \theta = \frac{2\pi}{3} \)
For each of the following right triangles, determine the exact value for \(\sin \theta\), \(\cos \theta\), and \(\tan \theta\).

1. \[ \sin \theta = \frac{3}{5} \quad \cos \theta = \frac{4}{5} \quad \tan \theta = \frac{3}{4} \]
2. \[ \sin \theta = \frac{9}{15} \quad \cos \theta = \frac{8}{15} \quad \tan \theta = \frac{9}{8} \]
3. \[ \sin \theta = \frac{21}{30} \quad \cos \theta = \frac{21}{30} \quad \tan \theta = \frac{21}{21} \]
Unit Circle, Trigonometric Functions, and Periodicity
Part 1 (Accommodated)
The first part of the test will be completed without a unit circle. Once you have turned in this section, you will receive the second part of the test with a unit circle. You may use a calculator for this portion of the test.

- Radian: the measure of a central angle that spans an arc whose length is equal to the length of the radius.

- Write the proportion used for changing from radians to degrees \( \frac{\pi}{180} \)

- Write the proportion used for changing from degrees to radians \( \frac{180}{\pi} \)

Convert each of the following radian measures to degrees.

1. \( \frac{\pi}{2} \)
2. \( \frac{6\pi}{7} \)
3. \( \frac{11\pi}{6} \)
4. \( \frac{3\pi}{2} \)
5. \( \pi \)
6. \( \frac{\pi}{9} \)
7. \( 2\pi \)
8. \( \frac{5\pi}{8} \)
9. \( \frac{8\pi}{11} \)

Convert each of the following degree measures to radians.

1. \( 55^\circ \)
2. \( 75^\circ \)
3. \( 100^\circ \)
4. \( 410^\circ \)
5. \( 290^\circ \)
6. \( 30^\circ \)
7. \( 200^\circ \)
8. \( 310^\circ \)
9. \( 97^\circ \)
- Period of a function: the distance required for the function to complete one full cycle

- Period of sine: $2\pi$

- Period of cosine: $2\pi$

Tell whether or not each of the following graphs are periodic.

A. Periodic
B. Not Periodic

A. Periodic
B. Not periodic
A. Periodic

B. Not periodic

A. Periodic

B. Not Periodic
Unit Circle, Trigonometric Functions, and Periodicity
Part 2 (Accommodated)
You may NOT use a calculator for this section of the test.
Use the given unit circle to answer the following section’s questions.
Give the trigonometric ratio for each of the following:

\[
\sin \Theta = \frac{y}{r} \quad \cos \Theta = \frac{x}{r} \quad \tan \Theta = \frac{y}{x}
\]

Calculate \(\sin \Theta\), \(\cos \Theta\), and \(\tan \Theta\) for each of the following radian or degree measures.

1. \(\Theta = 225^\circ\)

2. \(\Theta = \frac{5\pi}{3}\)

3. \(\Theta = 90^\circ\)

4. \(\Theta = \frac{\pi}{6}\)

5. \(\Theta = 270^\circ\)

6. \(\Theta = \frac{2\pi}{3}\)
For each of the following right triangles, determine the exact value for \( \sin \theta \), \( \cos \theta \), and \( \tan \theta \).

\[\begin{array}{c}
\sin \theta = \\
\cos \theta = \\
\tan \theta = \\
\end{array}\]

\[\begin{array}{c}
3 \\
-4
\end{array}\]

\[\begin{array}{c}
5
\end{array}\]

\[\begin{array}{c}
9 \\
15
\end{array}\]

\[\begin{array}{c}
8
\end{array}\]

\[\begin{array}{c}
30 \\
21
\end{array}\]

\[\begin{array}{c}
19
\end{array}\]

\[\sin \theta = \\
\cos \theta = \\
\tan \theta = \\
\]
Test Key

- the measure of the central angle that spans an arc whose length is equal to the length of the radius

\[ \frac{\pi}{180} \]

\[ \frac{180}{\pi} \]

1. 90°
2. 154.29°
3. 330°
4. 270°
5. 180°
6. 20°
7. 360°
8. 112.5°
9. 130.9°

1. \[ \frac{11\pi}{36} \]
2. \[ \frac{5\pi}{12} \]
3. \[ \frac{5\pi}{9} \]
4. \[ \frac{41\pi}{18} \]
5. \[ \frac{29\pi}{18} \]
6. \[ \pi/6 \]
7. \[ \frac{10\pi}{9} \]
8. \[ \frac{3\pi}{8} \]
9. \[ \frac{97\pi}{180} \]

Period of a function: the distance required for the function to complete a full cycle

Period of sine: \( 2\pi \)
Period of cosine: \( 2\pi \)

\[ \begin{align*}
A & \quad \sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x} \\
B & \quad \sin \theta = \frac{3}{5} \quad \cos \theta = \frac{-y}{5} \quad \tan \theta = -\frac{3}{4} \\
B & \quad \sin \theta = \frac{8}{15} \quad \cos \theta = \frac{3}{5} \quad \tan \theta = \frac{8}{9} \\
A & \quad \sin \theta = \frac{19}{30} \quad \cos \theta = \frac{21}{30} \quad \tan \theta = \frac{19}{21}
\end{align*} \]
The answer of this question number 1 is shown on page number 5.

\[
\frac{\pi}{2} \\
\frac{\pi}{4} \\
\frac{\pi}{6} \\
\frac{\pi}{8}
\]

I have 3 sins and 3 cosines. I need to find the values of each. Let's list the values of sine.

\[
\sin \theta = 0 \\
\sin \theta = \frac{1}{2} \\
\sin \theta = \frac{\sqrt{2}}{2} \\
\sin \theta = \frac{\sqrt{3}}{2} \\
\sin \theta = 1
\]

\[
\cos \theta = 1 \\
\cos \theta = \frac{\sqrt{3}}{2} \\
\cos \theta = \frac{\sqrt{2}}{2} \\
\cos \theta = \frac{1}{2} \\
\cos \theta = 0
\]
Day 7:
Daily Lesson Plan

Day: (Day 7)
Objectives:
- TSW
  - Create a word problem. (DOK 4)
  - Apply concepts of trigonometric functions and right triangles to real life situations. (DOK 4)
  - Show these ideas as a presentation to the class. (DOK 2)

Materials:
- Popsicle stick cup with students’ names written on them
- Student project visual aids
  - May include computer, Apple TV, or whiteboard
- Project rubric
- Pencil/pen to write on rubric
- Cars to put in Parking Garage on the way out
- Pencil/pen to write on cars
- Parking Garage pocket board

Opening (Set): 2 minutes
- You all did a great job on your test yesterday!
- I know all of you worked hard on your projects, because we will be presenting them today!
- We will go in random order as I pull your names from the Popsicle stick cup. If I pull a person from your pair, that pair will present.
- When you come up to present, each individual will hand me a rubric with only his/her name on it. You will be given individual grades, even if you are in a group with other people.
- I want everyone to pay attention to each presentation. Ask any questions about the word problems that you can think of. This will be a learning experience for the audience, the presenters, and me.
- Let’s begin!

Learning Tasks (Procedures): 46 minutes

I. Begin the presentations.
   a. While someone is at the front of the classroom presenting, I would suggest you write down any mathematical notes you think you should know.
   b. The teacher will pull a name from the Popsicle stick cup.
   c. A rubric should be handed in with all the names of the people who worked on the project.
   d. That individual/pair/group will present.
e. This process will be repeated until time is running out for class. The rest of the students should take the entire class tomorrow for presenting as well.

Closure: 2 minutes

- All of you did such a great job presenting and such a great job observing!
- To end class, I am going to hand out cars. I want each student to write down your favorite presentation of the day and why it was your favorite. When the bell rings, place the cars in the parking garage on your way out.
- Those who did not present today will present tomorrow. Have a great rest of the day!

Differentiated Instruction:

- **Enrichment:** These students will be firmly graded in terms of their word problem material and visual aid. Any mistake in their math will result in points taken off.

- **Intervention:** These students will be counted off less for any mistake made in the math of their word problem. Their visual aid and presentation points will be graded firmly.

- **Accommodation:** These students will not be graded as harshly on any level of the project. Their content knowledge does not need to be high enough to teach a lesson to the class. Their presentation skills may also be taken at the first level for full credit. These students will also be in groups with the advanced or regularly performing students.
Day 8:
Daily Lesson Plan

Day: (Day 8)

Objectives:
- TSW
  - Create a word problem. (DOK 4)
  - Apply concepts of trigonometric functions and right triangles to real life situations. (DOK 4)
  - Show these ideas as a presentation to the class. (DOK 2)

Materials:
- Popsicle stick cup with students’ names written on them
- Student project visual aids
  - May include computer, Apple TV, or whiteboard
- Project rubric
- Pencil/pen to write on rubric
- Cars to put in Parking Garage on the way out
- Pencil/pen to write on cars
- Parking Garage pocket board

Opening (Set): 2 minutes
- You all did a great job presenting yesterday!
- Those of you who did not present yesterday will be presenting today.
- We will go in random order as I pull your names from the Popsicle stick cup. If I pull a person from your pair, that pair will present.
- When you come up to present, each individual will hand me a rubric with only his/her name on it. You will be given individual grades, even if you are in a group with other people.
- I want everyone to pay attention to each presentation. Ask any questions about the word problems that you can think of. This will be a learning experience for the audience, the presenters, and me.
- Let’s begin, again!

Learning Tasks (Procedures): 46 minutes

II. Begin the presentations.
   a. While someone is at the front of the classroom presenting, I would suggest you write down any mathematical notes you think you should know.
   b. The teacher will pull a name from the Popsicle stick cup.
   c. A rubric should be handed in with all the names of the people who worked on the project.
   d. That individual/pair/group will present.
   e. This process will be repeated until all students have presented or until time has run out for class, whichever comes first.
Closure: 2 minutes

- *All of you did such a great job presenting and such a great job observing!*
- To end class, I am going to hand out cars. I want each student to write down your favorite presentation of the day and why it was your favorite. When the bell rings, place the cars in the parking garage on your way out.
- *We have reached the end of our unit. I am so impressed with how each of you has performed. We will begin our next unit tomorrow. Have a great rest of the day!*

Differentiated Instruction:

- **Enrichment:** These students will be firmly graded in terms of their word problem material and visual aid. Any mistake in their math will result in points taken off.

- **Intervention:** These students will be counted off less for any mistake made in the math of their word problem. Their visual aid and presentation points will be graded firmly.

- **Accommodation:** These students will not be graded as harshly on any level of the project. Their content knowledge does not need to be high enough to teach a lesson to the class. Their presentation skills may also be taken at the first level for full credit. These students will also be in groups with the advanced or regularly performing students.